in very good agreement with the present experimental results. The magnetic moment of the ground state of Tb^{159}, which is also probably a [411, $\frac{3}{2}$+] Nilsson state, was measured by Baker and Bleaney and found to be (1.90±0.05) nm[16] (if Lindgren's values[17] of $\langle 1/r^3 \rangle$ are used) which is very close to the present experimental value of the magnetic moment of the 103-keV level.

Gilat and Nowik calculated the effective electric field gradient at the Eu nucleus in EuIG, associated with the orientation of the orbital wave functions produced by the exchange interaction through the spin-orbit coupling. Their calculated value was −55 Mc/sec per barn at 20°K. For the ground state $Q=2.4$ b and therefore a value of about −130 Mc/sec is expected for eqQ_0, neglecting other possible contributions to the electric field gradient.[4] The present experimental upper limit of 200 Mc/sec for the absolute value of eqQ_0 is therefore not in contradiction with the theoretical prediction.

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Shapes of Allowed and Unique First-Forbidden β-Ray Spectra: In^{114}, K^{42}, Rb^{86}, Sr^{90}, and Y^{90}

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The β-ray spectra of In^{114}, K^{42}, Rb^{86}, Sr^{90}, and Y^{90} (ground-state transitions) have been measured with the Heidelberg double-lens spectrometer. The allowed spectrum of In^{114} was found to have a purely statistical shape; the coefficient b of a possible b/W term turned out to be $b=(0.5\pm2.2)\times10^{-2}mc^2$. The unique first-forbidden spectra of K^{42}, Rb^{86}, Sr^{90}, and Y^{90} were found to show very small but definite deviations from the simple unique shape.

I. INTRODUCTION

According to the accepted theory of weak interaction, the shape of allowed spectra must be "allowed" or statistical, unless second-order terms contribute significantly. These second-order terms are (1) "regular" twice-forbidden contributions and (2) weak-magnetism terms.[1] Both kinds of terms have been observed. The largest deviation from the statistical shape was found[2–9] in the decay of P^{32} and can fully be explained[10,11] by the high ft value of this decay.

In the case of unique first-forbidden spectra, one expects a "unique" shape unless there are considerable (1) "regular" third-forbidden contributions and (2) weak magnetism terms. The situation is, however, more complex.[12]

Previous work at laboratories in Heidelberg has verified the statistical shape for a number of allowed β-ray spectra.[3,13–18] Similar results were obtained by other groups.[2,7,8,19–21] These results are incompatible with the work of Langer and co-workers[5,22–24] who have been reporting b/W type deviations from the statistical shape, with b values centered around +0.3. Such large

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deviations were also reported by two other groups.[25,26] Nevertheless, there is no satisfactory explanation for such a deviation.

Much less work has been reported for the study of the unique first-forbidden spectra. Several measurements indicated small or vanishing deviations from the unique shape.[7,27–29]. On the other hand, Langer and co-workers[5,30–33] have reported b/W type deviations with large b values.

It was the purpose of the present work to remeasure the allowed spectrum of In^{114} and to measure the unique first-forbidden spectra of K^{42}, Rb^{86}, Sr^{90}, and Y^{90} (ground-state transitions). In^{114} is particularly suitable as a test for the shape of an allowed spectrum because the ft value is low and the experimental conditions are favorable. The various unique transitions cover a fairly wide range of atomic numbers and maximum energies and in this way allow an, at least crude, survey of the field. The ft values corrected for forbiddenness are normal and the experimental conditions are good.

II. METHOD AND RESULTS

In^{114m} and Rb^{86} were produced by neutron irradiation of isotopically enriched In metal and natural RbCl, respectively, in the reactor over a period of four weeks with 2×10^{13}n cm^{-2} sec^{-1}. K^{42} was produced by a 7-h bombardment of KCl, isotopically enriched in K^{41}, in the Heidelberg cyclotron. Sr^{90} and Y^{90} were obtained as a carrier-free solution. No chemistry was done to separate the activities from each other.

The radioactive material was evaporated *in vacuo* on Al backings of 0.24 mg/cm^2 (In^{114}, K^{42}, and Rb^{86}) or on a backing of 0.33 mg/cm^2 mica plus evaporated conductive metal layer ($Sr^{90}-Y^{90}$). The source thicknesses, as determined with an alpha gauge,[34] were: In^{114}, 0.16 mg/cm^2; K^{42}, 0.17 mg/cm^2; Rb^{86}, 0.12 mg/cm^2; and $Sr^{90}-Y^{90}$, 0.09 mg/cm^2. With these source and backing thicknesses, no distortions of the spectra can occur in the energy region under investigation. The spectra were measured with the Heidelberg double-lens spectrometer, which is particularly suitable for spectral-shape measurements. The techniques were previously described.[6,13,35,36]

For the evaluation of the experimental data the tables of Dzhelepov and Zyrianova[37] were used which include screening and finite size effect. In this notation one has $L_0\equiv1$. A check with new calculations of Bühring[12] in the case of Y^{90} led to almost identical results.

From the measured data, the allowed shape factor

$$C_0(W)=N(p)/[F(Z,W)p^2(W_0-W)^2] \quad (1)$$

was computed in the case of the allowed transition (In^{114}) while in the case of the unique first-forbidden transitions (K^{42}, Rb^{86}, Sr^{90}, Y^{90}) the corrected shape factor

$$C_1(W)=N(p)/[F(Z,W)p^2(W_0-W)^2(q^2+9L_1)] \quad (2)$$

was computed; the symbols have the usual meaning.[37,38] Note, however, that L_1 in Eq. (2) is the L_1 of Ref. 37

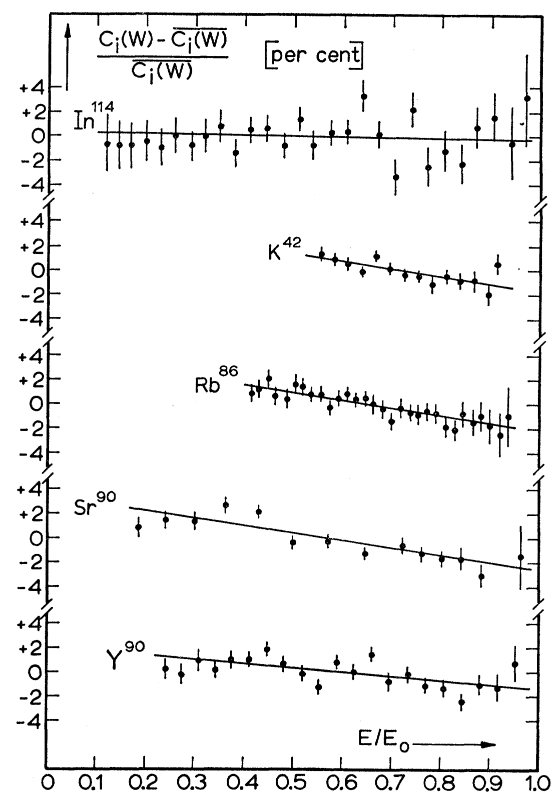

FIG. 1. Experimental results of the shape-factor measurements. For each isotope, the quantity $[C_i(W)-\langle C_i(W)\rangle_{av}]/\langle C_i(W)\rangle_{av}$, i.e., the relative shape-factor deviation, is plotted against E/E_0 (kinetic energy in units of the maximum kinetic energy). The shape factors $C_i(W)$ are defined by Eqs. (1) and (2) ($i=0$: allowed case; $i=1$: unique first-forbidden case). The solid straight lines are determined by least-square fits according to Eqs. (4). The slope of these lines measures the deviation from the expected shape factors Eq. (1) or (2), respectively.

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TABLE I. Results of the present measurements.

Isotope	Type of the decay	$\log ft$ or $\log f_1 t$	E_0[MeV]	$a[(mc^2)^{-1}]$	$b[mc^2]$
In^{114}	allowed	4.4	1.988 ± 0.005	$(-1.5\pm3.0)\times10^{-3}$	$(0.5\pm2.2)\times10^{-2}$
K^{42}	1st unique	8.4	3.524 ± 0.012	$(-10\pm4)\times10^{-3}$	
Rb^{86}	1st unique	8.4	1.774 ± 0.005	$(-17\pm2)\times10^{-3}$	
Sr^{90}	1st unique	8.3	0.546 ± 0.002	$(-54\pm19)\times10^{-3}$	
Y^{90}	1st unique	8.3	2.284 ± 0.005	$(-7.2\pm3.2)\times10^{-3}$	

suitable for $F(Z,W)$ of the same reference. As this L_1 (Table 24 of Ref. 37) is tabulated in large intervals only, it was interpolated (and, in the case of K^{42}, also extrapolated) with the help of the "old" L_1 (Table 22 of Ref. 37).

In order to look for deviations from the expected shape, least square fits of the following forms were performed:

$$C_0(W)=\text{const}(1+b/W) \tag{3}$$

and

$$C_i(W)=\text{const}(1+aW), \tag{4}$$

with $i=0$ (allowed case) and $i=1$ (unique first-forbidden case).

Figure 1 shows the results of the shape-factor measurements. For each isotope the quantity $[C_i(W)-\langle C_i(W)\rangle_{\text{av}}]/\langle C_i(W)\rangle_{\text{av}}$ is plotted against E/E_0 (kinetic energy in units of the maximum kinetic energy), with $\langle C_i(W)\rangle_{\text{av}}$ as the weighted mean of all $C_i(W)$ values in the measured region. This region varies from isotope to isotope because, in the case of K^{42} and Rb^{86}, an inner group or, in the case of Y^{90}, the parent activity masks the low-energy part of the unique spectrum. The In^{114} spectrum was corrected for the very weak inner group.

Table I lists the results. The stated errors in the maximum energy E_0 are estimated figures which include the calibration error. The errors in a and b are standard deviations of the least-square fits. In the case of Sr^{90} the error of the Y^{90} subtraction is included.

III. DISCUSSION

The measurements of this work clearly show that the In^{114} β spectrum has a statistical shape. This is in agreement with the older Heidelberg[15] and Ames[7] findings but in striking disagreement with the result reported by Langer and co-workers.[5] Combined with earlier experiments on other nuclei[2,3,8,13–21] (cf. Sec. I), one can safely conclude that, besides very small weak magnetism[1] and twice forbidden terms, the spectrum of an allowed β transition is statistical.

In the same way no b/W term has been observed in this work for the unique first-forbidden spectra of K^{42}, Rb^{86}, Sr^{90}, and Y^{90}. This is in agreement with earlier work[7,27,29] in the case of Y^{90}, but again in contradiction to the work of Langer *et al.*[5,30,33] in the case of Rb^{86} and Y^{90}.

It has been found in this work that the unique forbidden spectra of K^{42}, Rb^{86}, Sr^{90}, and Y^{90} do not exactly show the previously expected shape, even when screening and finite-size effect are properly taken into account. There are now more elaborate theoretical treatments available[39–41] which include weak magnetism and other higher-order effects. In this case the corrected shape factor is no longer given by Eq. (2), and the use of Eq. (4) to fit the experimental data is essentially justified by its simplicity only. One may hope to learn more about the higher-order terms when combining the more elaborate general theoretical expressions with specified nuclear-model calculations.

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